Physics of the guitar at the Helmholtz and first top-plate resonances*

Ian M. Firth

School of Physical Sciences, University of St. Andrews, St. Andrews, Fife, Scotland
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The action of the guitar has been studied in detail in the vicinity of the Helmholtz air resonance and the first resonance of the top plate. Measurements of the input mechanical admittance, the output sound pressure, and their phases suggest an analogous acoustical circuit for the guitar identical with that used to describe the action of a loudspeaker in a bass-reflex enclosure. Below the Helmholtz resonance sound radiated from the rose and from the top plate are out of phase. As a result sound radiated from the guitar is not enhanced by the rose. Above this resonance sound output from the rose and top plate are more nearly in phase with a resulting enhancement of radiated sound.

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INTRODUCTION

In this study, the behavior of the guitar in the vicinity of its first two resonances has been studied in detail. The two resonances are the Helmholtz air resonance (A0) at about 90 Hz and the first top-plate resonance (T1) at about 180 Hz. The guitar was chosen for this study because it has one air port (the rose) and the bridge is an integral part of the construction of the top plate. Although more information is available on the action of the violin, the two ff holes and the unattached bridge would make results more difficult to interpret unambiguously. In investigations of the radiation from an instrument directional patterns are also involved. Most modes of vibration of the violin are asymmetrical,[1] whereas, those at A0 and T1 in the guitar are symmetrical[2] thereby making measurement of radiation and its interpretation considerably easier.

Preliminary studies to record the modes of vibration of the guitar were undertaken using powder-pattern techniques. Measurements of the input mechanical admittance at the bridge and the radiated sound from the guitar at the rose and at the top plate, and their phases, were made in the range of A0 and T1. An analogous acoustical circuit for the action of the guitar at A0 and T1 has been inferred from measurements. In this frequency range the action of the guitar can be described in the same way as a loudspeaker (the top plate) in a bass-reflex enclosure (the volume of the guitar, with the rose acting as the open port). The guitar used in these studies was a Levin LG 17 in the white: measurements of radiation were made in anechoic conditions in the range 70–260 Hz.

I. METHODS AND RESULTS

A. Preliminary investigations of modes of vibration

Before undertaking detailed measurements in the range of A0 and T1 it was necessary to identify some of the modes of vibration of the freely suspended guitar. Measurements on another Levin LG 17 guitar using interference holographic techniques served as a basis for these preliminary measurements, Fig. 1. The first and second top-plate modes T1 and T2 occurred in this guitar at 148 and 236 Hz.

Three types of measurements were made.

1. Plate-tone testing

The guitar was mounted vertically in an anechoic room, being clamped at the neck and resting on a plastic foam pad at the button. A minishaker, on which was mounted a B&K impedance head, type 8000, was applied to the bridge at the low E string. The force of excitation was held constant by means of the compression circuit of the B&K heterodyne analyzer, type 2010, and the guitar was vibrated as the frequency of vibration was swept. The output SPL from the guitar was recorded by a 1-in. B&K microphone, type 2619, placed at 1 m from the top plate of the guitar. SPL was measured within a bandwidth of 3 Hz about the drive frequency. Figure 2(a) shows sound output versus frequency for a constant force of excitation.

In order to locate the Helmholtz resonance of the guitar, two tests were employed. Firstly, the rose was covered with a dense cardboard circle sealed with adhesive tape to the top plate and plate-tone testing repeated: the peak at 90 Hz disappeared, those at 180 and 205 Hz remained, but their frequencies changed to 177 and 205 Hz.

Secondly, a probe microphone was inserted through the open-rose well into the body of the guitar when the

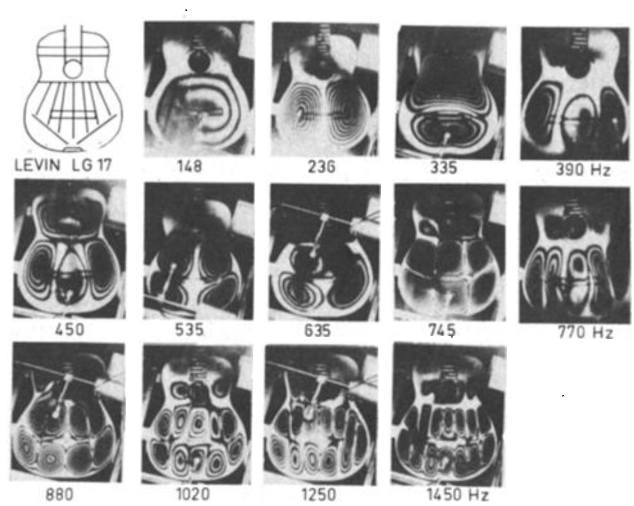

FIG. 1. Time average interference holograms of modes of vibration of the top plate of a Levin LG 17 Guitar.

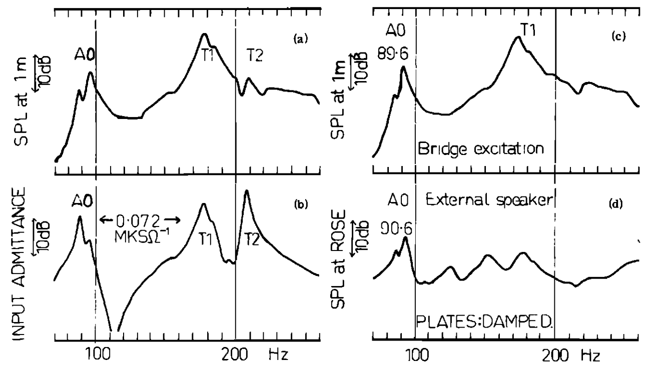

FIG. 2. (a) Sound pressure level at 1 m for LG 17 excited at low-E-string position on bridge. (b) Input admittance at low-E-string position on bridge. (c) Sound pressure level at 1 m; guitar excited at the center of the bridge. (d) Sound pressure level in the belly of the guitar when an external loudspeaker is swept in frequency. Top and back plates of guitar are heavily damped. Force of excitation is constant in (a), (b), and (c).

top and back plates of the guitar were heavily damped by wrapping blankets around the instrument, except near the rose. Figure 2(d) shows the response of the guitar air volume when an external loudspeaker was swept in frequency: the 90-Hz resonance is recognized as the Helmholtz resonance A0, and the 180- and 205-Hz resonances as due to the top plate.

2. *Input impedance measurements across the top plate*

The output voltage from the impedance head which is proportional to acceleration was integrated to give a signal proportional to the velocity of the plate at the point of excitation. This voltage is proportional to the input mechanical admittance when the force of excitation is held constant. Figure 2(b) shows the input admittance of the top plate at the bridge at the low E string.

In order to identify the pattern of vibration of the top plate which gives rise to the peaks of input admittance at 90, 180, and 205 Hz, input admittance was measured at these frequencies by moving the vibrator and impedance head transversely at the bridge and lengthwise along the center line of the guitar top plate. Admittance measured during these traversals is shown in Fig. 3.

The standing waves which were measured show that the first top-plate resonance T1 occurs at 179 Hz and the second T2 at 205 Hz. The T1 mode is excited again at 90 Hz, the frequency of the Helmholtz resonance.

3. *Powder patterns*

Powder patterns of vibrational modes were made by vibrating the guitar over the loudspeaker. The guitar was mounted on small foam-plastic blocks. Glitter[3] was sprinkled on to the uppermost plate and the frequency of the loudspeaker adjusted to a resonance of that plate. The blocks of foam were moved to nodal positions. Figure 4 shows powder patterns formed on top and back plates.

The pattern recorded on the top plate at 90 Hz is similar in shape to T1 at 180 Hz, as it should be.

There is one important difference, however: there is vibration of the top plate outside the nodal line at 180 Hz, but the whole area of vibration of the top plate is contained within the glitter line at 90 Hz. This indicates that the ribs are not moving to any extent at 90 Hz and that the top plate is being driven essentially by the motion of the air inside the body.

The mode T2 could not be obtained by vibrating over the loudspeaker. A coil transducer fixed to the bass side of the bridge excited the T2 mode at 166 Hz, lower in frequency than previously recorded in Fig. 2 because of mass loading by the coil. Motion of the back plate occurs also at 89 Hz, but the first true back-plate mode occurs at 250 Hz. Other back-plate modes are at 391, 405, and 447 Hz.

Because the resonances T1 and T2 overlap to a considerable extent all subsequent experiments were undertaken with the point of excitation of the top plate moved to the center of the bridge. The T2 mode was not excited therefore, and the behavior of the guitar in the region of A0 and T1 was simplified. Figure 2(c) shows the sound radiated when the guitar is excited at the center of the bridge.

B. Input admittance in range of A0 and T1

The input admittance to the freely suspended guitar was measured at the center of the bridge. The phase ϕ_{V-F}, of the voltage representing the velocity of the point of excitation was measured with respect to that representing force. A Farnell phase meter was used and phase was recorded on a chart. A correction to the measurement of phase is required to take account of the electrical circuit of the measuring equipment. The correction is obtained by repeating admittance measurements with a brass weight fixed to the impedence head. This experiment gives a calibration of phase (velocity lags force, $\phi_{V-F} = +90°$) and of admittance (admittance = velocity/force$= -j/\omega m$). Input admittance to the free guitar (Y) and the corrected phase (ϕ_{V-F}) are shown in Fig. 5(a), and in Fig. 5(b) for the

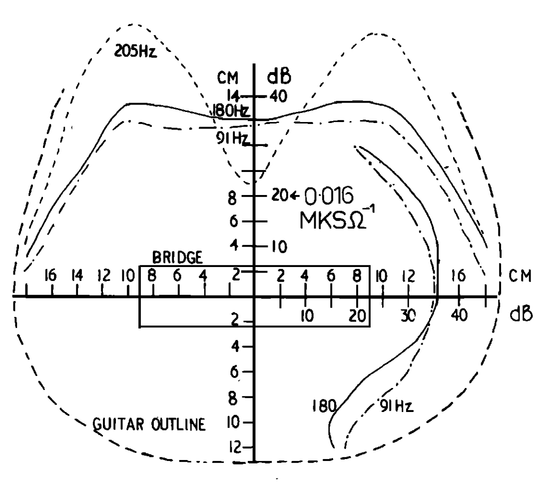

FIG. 3. Input admittance to the top plate of LG 17 guitar. Admittance is measured along the line of the bridge and along the midline of the guitar at A0 (90 Hz), T1 (180 Hz), and T2 (205 Hz).

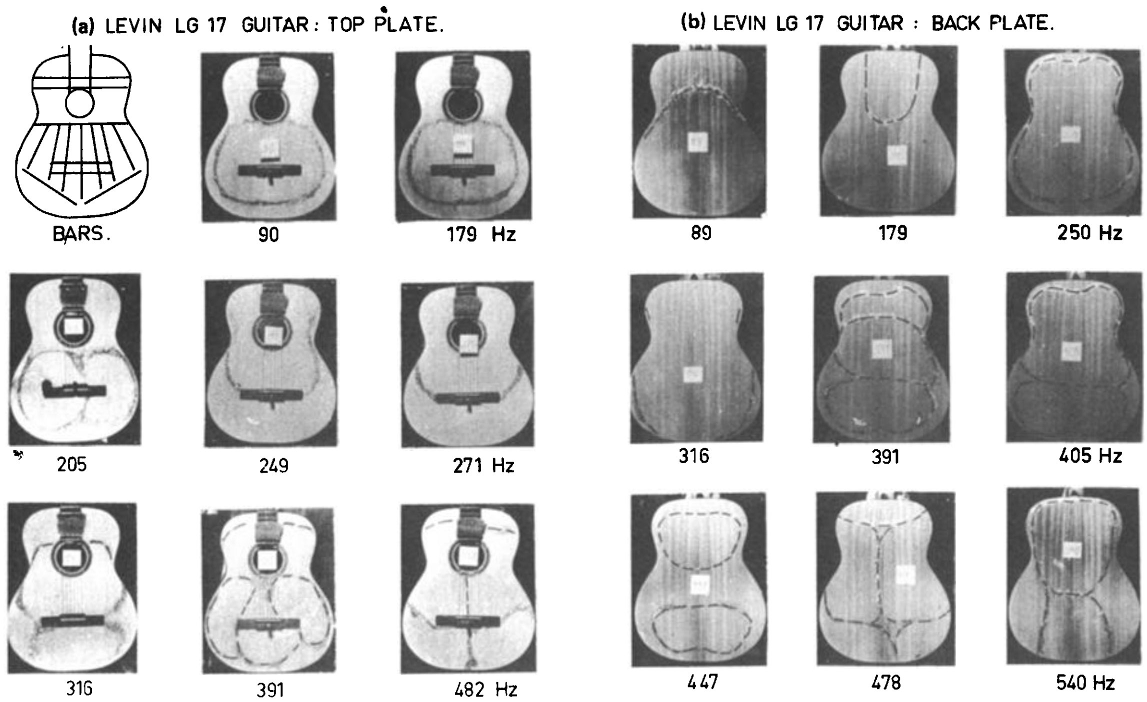

FIG. 4. (a) Chladni powder patterns of top plate of guitar. (b) Chladni powder patterns of back plate of guitar.

guitar when the rose is closed. Table I summarizes measurements of frequency and Q at A0 and T1.

Considering the results in Figs. 2(d), 5(a), and 5(b) it is possible to suggest an analogous acoustical circuit for the guitar with respect to the point of excitation at the bridge. At the resonances of admittance of Fig. 5(a), ϕ_{V-F} is zero so that A0 (89.6 Hz) and T1 (178.6 Hz) are internal zeros of the circuit. The antiresonance at 110 Hz is an internal pole. Figure 2(d) shows that when T1 is heavily damped the air resonance A0 rises slightly in frequency to 90.6 Hz. Figure 5(b) indicates that when the rose is closed the resonance frequency of T1 is reduced to 163.6 Hz.

An analogous acoustical circuit[4] which has a similar response at its input terminals as Fig. 5(a) in the vicinity of A0 and T1 is shown in Fig. 6. This is very similar to the circuit which is used to describe the action of a loudspeaker in a bass-reflex enclosure.[5]

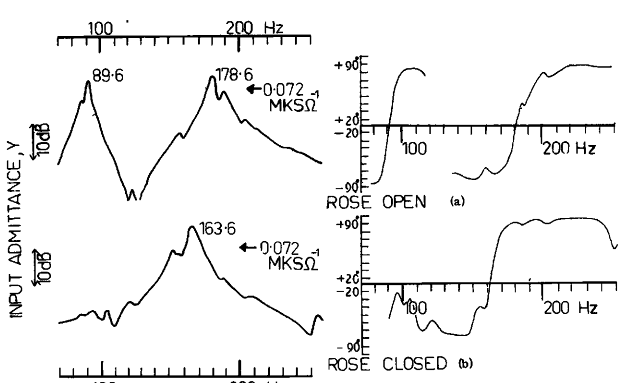

FIG. 5. Input addmittance and its phase with respect to force of excitation measured at the center of the bridge with (a) the rose open and (b) the rose closed.

The circuit is drawn so that the input terminals represent a point of excitation at the center of the bridge. There are two parts to the circuit; the series elements represent the whole wooden guitar at the first resonance of the top plate; the parallel elements represent the action of the air in the body of the guitar at the Helmholtz resonance.

From the measurements listed in Table I the input impedance to the equivalent circuit (Fig. 6) can be calculated from Fig. 5(a). The values of a series combination of M, C, and R which would make up this input impedance are $M = 0.53$ kg, $C = 6\ 10^{-6}$ mN^{-1}, and $R = 10$ mks Ω at A0, and $M = 0.31$ kg, $C = 2.5\ 10^{-6}$ mN^{-1} and $R = 9.5$ mks Ω at T1.

The two parts of the equivalent circuit of the guitar cannot be separated in a completed guitar. Measurements on the top plate are always influenced by the presence of the circuit of the Helmholtz resonance. For instance, when A0 is inactivated by closing the rose (open circuiting the rose elements) the circuit representing the action of the top plate is still loaded

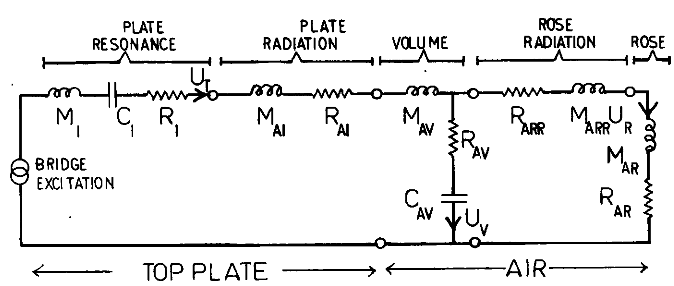

FIG. 6. Analogous acoustical circuit which describes the action of the guitar in the vicinity of A0 and T1. The point of excitation (i.e., the input terminals) is the center of the bridge.

TABLE I. Resonance frequencies and Q's of guitar.

		f (Hz)	Q
Air (A0) resonance	Free guitar	89.6	30
	Top-plate damped	90.6	22
Top-plate (T1) resonance	Free guitar	178.6	37.2
	Rose closed	163.6	21

by some of the elements of the air resonance M_{AV}, R_{AV}, and C_{AV}. Therefore, the true resonance frequency of the series elements of the top plate along (M_1, C_1, and R_1) can never be measured in the complete guitar (except perhaps *in vacuo*). The true Helmholtz resonance can be investigated if the walls are immobilized, perhaps by burying the guitar in sand. The experiment used to obtain Fig. 2(d) will not completely immobilize the walls but shows correctly, nevertheless, that when the walls are damped the Helmholtz frequency is less than in the free guitar.

The reactances of the two parts of the analogous circuit are shown diagramatically in Fig. 7 when all the resistances are zero. The velocity response U of the circuit to a constant force of excitation is also indicated compare with Fig. 5(a).

In the completed guitar it is possible to modify, quite readily, certain of the circuit elements of Fig. 6. M_1 can be altered by mass loading the bridge with metal weights; M_{AV}, R_{AV}, and C_{AV} can be changed by filling the volume with blocks of wood; R_{ARR}, M_{ARR}, M_{AR}, and R_{AR} can be altered by changing the area and shape of the rose opening. Measurements on component parts of a guitar as it is being made should enable most of the elements in the analogous circuit to be measured independently.[5] This would be the best way to obtain a complete description of the circuit in the region of A0 and T1.

C. Sound output and its phase

The guitar was mounted and excited as before and the sound pressure was measured (a) at the center of the rose, (b) adjacent to the impedance head at the center of the bridge and close to the bridge, and (c) at 1 m from the rose and the center of the bridge in the midplane of the guitar. The sound pressure was measured with a $\frac{1}{2}$-in. B&K microphone to which was fixed a 4-mm capillary probe. The phase of the sound pressure was measured with respect to the force of excitation for the three positions. The force of excitation was kept constant as frequency was swept from 70 to 260 Hz.

At positions (a) and (b) a correction was applied to the measurement of phase to take into account the phase lag (which varies with frequency) due to the capillary probe and connections of the electrical measuring equipment. For position (c) a further correction was applied to compensate for the time delay of waves traveling from the guitar. This correction effectively moves the point of measurement to the plane of the top plate.

Phase changes by more than a revolution. Which zero to adopt can be fixed as follows: The velocity of the top plate is in phase with the force of excitation at the first top-plate resonance, and the top plate acts then as a simple piston. The sound pressure measured at the top plate therefore leads force of excitation by 90° at T1. The SPL and corrected phase angle between sound pressure and force of excitation at the bridge are shown in Fig. 8 for the three positions of measurement.

At this point it is possible to link the action of the guitar described in Figs. 5(a) and 8 with the behavior of the analogous acoustical circuit of the guitar shown in Fig. 6. Referring to Fig. 6 the sound pressure at a point in the midplane a distance r_{av} away from the guitar is

$$P=(if\rho/2r_{av})(U_T e^{-ikr}-U_R e^{-ikr}) .$$

The distance r to rose and bridge is corrected for in phase measurements so that, referred to the plane of the top plate of the guitar

$$P\simeq(if\rho/2r_{av})(U_T-U_R)$$
$$=(if\rho/2r_{av})U_V,$$

where U_T is the complex rms volume velocity of the top plate at the region of the bridge (U_T is measured by Y), and U_R is the complex rms volume velocity at the rose. A negative sign is used for U_R because, at frequencies far below the air resonance an outward displacement of air by the top plate is canceled by an equal inward intake through the rose, and $U_V=U_T-U_R$ is the volume velocity necessary to compress and expand the air inside the guitar.

Consider now the behavior of the acoustical circuit and the guitar with rising frequency.

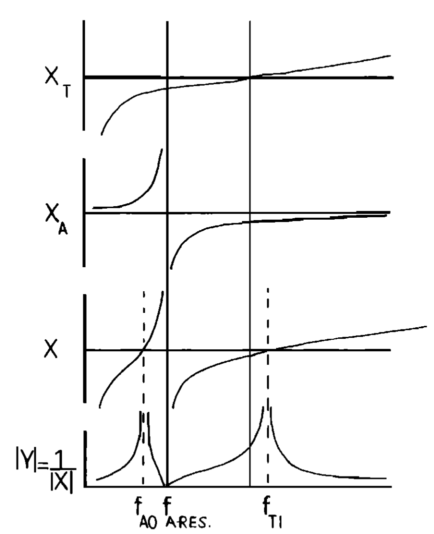

FIG. 7. Reactances of the series (X_T) and the parallel (X_A) parts of the analogous circuit. The input reactance ($X=X_T+X_A$) and the velocity response $|U|=|Y|=|X|^{-1}$ are shown.

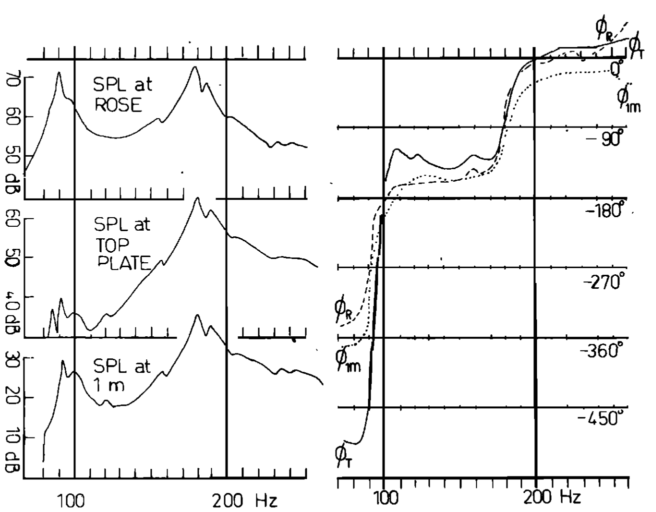

FIG. 8. Sound pressure level at rose, top plate, and 1 m, and the phase of pressure with respect to force of excitation versus frequency. ϕ_R—phase of pressure at rose: ϕ_T—phase of pressure at the center of bridge: ϕ_{1m}—phase of radiated pressure at 1 m corrected to the plane of the top plate.

1. Below f_{A0}

For frequencies well below f_{A0} the reactance of C_{AV} is very high and U_T becomes nearly equal to U_R. Radiation from the rose (proportional to $-U_R$) is out of phase with the radiation from the top plate (proportioned to U_T). Figure 8 confirms this prediction to be the case: ϕ_R differs by 180° from ϕ_T.

As a result of this phase difference radiation (measured here at 1 m) from the guitar at very low frequencies is not enhanced by the presence of the rose. The rose and the top plate behave like a dipole so that the SPL decreases by 12 dB per octave. As the reactance of C_{AV} is high, U_T decreases in this frequency range by a further 6 dB per octave so that the radiated SPL from the guitar should decrease by an overall 18 dB per octave below f_{A0}. Figure 8 shows that the radiated SPL from the guitar decreases more rapidly than that of the SPL at the rose, but careful measurements of the rates were not made.

At frequencies just below f_{A0}, Fig. 8 indicates that the radiated sound pressure and its phase follow closely that detected at the rose. Most radiation just below A0 arises from the rose, $U_V = (-U_R)$.

2. At the air resonance A0

Although input reactance is zero at f_{A0}, $(-U_R)$ will not be in phase with U_T. The phase of $(-U_R)$ is determined by the circuit elements in the part of the acoustical circuit which describes the air in the guitar. The behavior of the analogous circuit indicates that $(\phi_R - \phi_T)$ should be decreasing with increasing frequency. At A0 radiated sound (U_V) should still be mainly due to volume velocity in the rose $(-U_R)$.

These predictions are confirmed by Fig. 5(a) and 8. At A0, $\phi_T = -90°$, and leads the phase of Y by 90°, $\phi_Y = 0°$: $(\phi_R - \phi_T)$ is decraesing through the resonance as frequency increases; the radiated SPL at 1 m follows the SPL detected at the rose, and $\phi_{1\,m} \simeq \phi_R$.

3. Above f_{A0}

An antiresonance occurs above the frequency of A0, According to the analogous circuit of Fig. 6 the antiresonance is caused almost entirely by the parallel circuit representing the Helmholtz resonance. In the parallel circuit $-U_R$ is larger than U_T by a factor Q of this part of the circuit, so that radiation from the guitar still occurs principally from the rose in the region of the antiresonance, $U_V = -U_R$. Figure 8 confirms that the radiated sound pressure level (SPL) follows closely the SPL measured at the rose.

The analogous circuit also indicates that above the antiresonance the phase of sound pressure at the rose (proportional to $-U_R$) is nearly in phase with the pressure at the top plate (proportional to U_T), see Fig. 7. Figure 8 confirms that the phases of radiation from the guitar, and pressures at rose and top plate are nearly the same. There is, therefore, a resulting enhancement of radiated sound because of the collaboration of sound pressure from rose and top plate. In loudspeakers the amount of the increase in response for a bass-reflex enclosure is generally about 5 dB greater than a simple closed enclosure over a frequency range of one or two octaves. By measuring the radiated sound at 1 m when the rose is closed this enhancement can be shown to be present in the guitar (Fig. 9).

Finally, it is useful to compare the input admittance [Fig. 5(a)], SPL at rose and top plate, and radiated SPL (Fig. 8) in the vicinity of the simple first top-plate mode T1 (Fig. 4). Over a range of 40 Hz the shape of these parameters is identical indicating that the guitar is being excited in a single simple mode of vibration in which in the input impedance and the generated volume velocities of air change smoothly.

II. CONCLUSIONS

The modes of vibration of the guitar have been measured in the range 70–260 Hz. Experimental methods

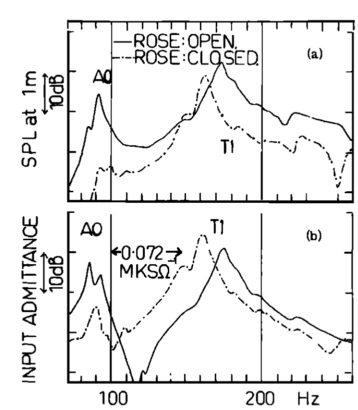

FIG. 9. (a) Radiated sound pressure level at 1 m when the rose is open and closed. (b) Input admittance to the center of the bridge when rose is open and closed.

are simple and enable the pattern of the modes to be easily interpreted.

An analogous acoustical circuit has been proposed for the guitar on the basis of input-admittance measurements at the center of the bridge. The circuit includes elements which describe the whole guitar at the resonance of the top plate and the air inside the body of the guitar at the Helmholtz resonance. The circuit is very similar to that used to describe a loudspeaker in a bass-reflex enclosure.

The circuit has been used to describe the action of the guitar in the range of the air resonance and the first top-plate resonance. The description is confirmed by measurements of sound pressure at rose and at top plate, and of the radiated sound pressure. The guitar behaves in this frequency range in the same way as a loudspeaker in a bass-reflex enclosure.

The acoustical advantages in using an open port, the rose, in the guitar might be considered as follows. As the top plate cannot be made thinner, or fixed to the ribs in a more flexible way the air resonance formed in the guitar extends the bass response of the instrument. The sharp cutoff of radiation below the air resonance implies that a maker has to adjust the frequency of this resonance with care.

These conclusions for the action of the guitar at its first two resonances should apply to all stringed instruments which have an air cavity, e.g., the lute, mandolin, violin, and viol.

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[1]E. Jansson, N-E. Molin, and H. Sundin, "Resonances of a Violin Body Studies by Hologram Interferometry and Acoustical Methods," Phys. Scr. 2, 243 (1970).

[2]E. Jansson, "Acoustical and Hologram Interferometric Measurements of the Top Plate Vibrations of a Guitar," Acustica 25, 95–100 (1971).

[3]Glo, 0.035 in. × 0.002 in. is made by Meadowbrook Inventions Inc., U.S.A., but is available in the U.K. from S. Fry and Co. Ltd., 23 Benwill Road, London N7 7BN.

[4]R. G. Meadows, *Electric Network Analysis* (Penguin Education, New York, 1972), Chap. 4.

[5]L. L. Beranek, *Acoustics* (McGraw-Hill, New York, 1954), Chap. 7, p. 241.